AIAA Journal
Vol. 42, No. 12, December 2004

Book Reviews

BOOK REVIEWS published in this section reflect the opinions of their individual authors. They are not necessarily the opinions of the Editors of this journal or of AIAA.

An Introduction to Nonlinear Finite Element Analysis

J. N. Reddy, Oxford University Press, New York, 2004, 463 pp, $114.50

Over the years a number of textbooks have presented monographs on nonlinear mechanics. Each of these books concentrated on specific issues such as heat transfer, fluid mechanics, or solid mechanics. The text under review attempts to present a unifying approach to nonlinear mechanics by incorporating each of these areas in the context of a computer-oriented finite element approach. The author accomplishes this in a volume of a little more than 460 pages, and so many of the specifics must be overlooked, but in general they can be found in the references listed at the end of each chapter.

In Chapter 1 the characteristics of nonlinearity are introduced using a simple pendulum incorporating the principle of the conservation of momentum. The resulting equation is a nonlinear differential equation. The method of solution requires numerical simulation first with the finite difference method and then with the finite element method. The concept of nonlinear softening and hardening is introduced. Some of the more pertinent nonlinear analyses texts are listed in the references. The title of Chapter 2, Finite Element Method: A Review, explains it all; it is a review of finite elements and it is successful in bringing into focus the weak form of integrals. A three-step procedure of constructing the weak form of the governing differential equation is presented. It is shown that the weak form of the governing differential equation leads to the variational problem. Shape functions are discussed. The weak form of a differential equation is extended into areas of continuum mechanics other than solid mechanics, i.e., fluid flow, heat transfer, and electrostatics, to name a few. Assemblies of elements are discussed, and numerical integration and coordinate transformation are reviewed. The use of FORTRAN is introduced in the development of some subroutines. This concept is followed throughout.

In Chapter 3, Heat Transfer and Other Field Problems in One Dimension, the author makes use of the one-dimensional heat transfer equation to introduce the technique of nonlinear equation solutions to finite element problems. Direct iteration and Newton–Raphson iteration techniques are presented, and some useful FORTRAN subroutines are included. As in Chapter 2, the author includes some interesting exercise problems and continues this throughout the text. Chapter 4, Nonlinear Bending of Straight Beams, makes use of the Euler–Bernoulli beam theory and presents a nonlinear development. The author includes some discussion of membrane locking as well as shear locking. This is brought about by the inconsistent shape functions associated with certain degrees of freedom. This idea is explained very well. Once again pertinent FORTRAN subroutines are presented.

Chapter 5 is entitled Heat Transfer and Other Field Problems in Two Dimensions. Again the weak form development, in which the three-step method is used, is initially stressed. The first step is to multiply the governing equation relating the internal dependent unknown and the known function of position in the domain and boundary with the weight function. In the second step, the integration by parts is carried out. The third step is to examine the boundary terms after the integration to characterize the coefficient of the weighting function along the boundary. The finite element technique with the appropriate interpolation functions is carried out in the context of the weak form equation considering heat transfer relations. As a final section in this chapter, the transformation to a master element is carried out with the use of the Jacobian matrix, which ends up with the result that the equivalent stiffness coefficients are evaluated using the Gauss quadrature. FORTRAN subroutines are listed to calculate certain element matrices.

Chapter 6, Nonlinear Bending of Elastic Plates and Shells, is the longest chapter of the text; more than 20 examples of nonlinear plate and shell theory are presented. The governing equations of the classical and first-order theories of plates and shells with the von Kármán strains are derived. The principle of virtual displacements is used to derive the weak forms, and the displacement finite element models are developed using the weak form. In the equation development, there is a close tie between the virtual work statement and the three weak forms established earlier. Thus, the background provided by Chapters 2–6 is illustrated. A great deal of the equation development can be found in other references; in fact the author lists 61 references. The most important parts of this chapter are the final nonlinear equations and the example problems showing the difference between nonlinear and linear solutions.

Chapter 7, Flows of Viscous Incompressible Fluids, reviews the governing equations of flow in an incompressible fluid. The introduction defines the major difference between the properties of solids and fluids, bringing out the fact that stress in a fluid is proportional to the time rate of strain. This is where viscosity is presented and the Reynolds number is defined. The introduction further states the relationship of the conservation of mass, momentum, and energy, which results in a set of nonlinear partial differential equations in terms of velocity, temperature, and pressure. The conservation of momentum equations lead to the Navier–Stokes equations. It is further stated that when temperature is not important, the energy equations are decoupled from the Navier–Stokes and continuity equations. This is an isothermal flow condition. The Eulerian description is used in formulating the governing equations, and the expressions for the conservation equations are presented and discussed. Incompressible fluid behavior is stressed. The author does a nice job of presenting the constitutive equations for a Newtonian fluid. Boundary conditions are discussed for the momentum equations (flow problem) and the energy equations (heat transfer problem). Three finite element models of the Navier–Stokes equations of the weak form relations are presented. The velocity–pressure model, the penalty model, which is similar to the Lagrange multiplier method (this method is described completely), and the least-square model are each discussed. Shortcomings of each model are presented. It would have been appropriate if the author had summarized the models at the end of the chapter, pointing out the differences. Several problems are solved and the results shown along with a listing of 44 references.

Chapter 8, Nonlinear Analysis of Time-Dependent Problems, starts in a somewhat unique way by presenting a discussion of the two stages involved in a finite element formulation of time-dependent problems. The first stage is the spatial approximation in which the space–time function is formulated using the procedure of static or steady-state problems while carrying all time-dependent terms in the formulation. The relation represents the spatial approximation of the primary function for any time. This step results in a set of ordinary differential equations in time for the nodal variables of the element. The second stage is the temporal approximation where the system of ordinary differential equations in time is further approximated in time. Often finite differences are used for the time derivatives, which lead to a set of algebraic equations for the primary variable time relation. This leads to general matrix differential equations. The matrices appearing in the algebraic equation can be functions of the primary variable making the differential equations nonlinear. Thus, at the end of the two-stage approximation, a continuous spatial solution at discrete time intervals is obtained. The remainder of the chapter studies the time approximation schemes and their stability and accuracy. The two-step procedure is followed in several problems, e.g., nonlinear heat conduction, bending of plates, and viscous incompressible flows. The author nicely describes the numerical techniques associated with the solution to parabolic and hyperbolic numerical algebraic differential equations. Pertinent FORTRAN statements are presented for the development of important matrices.

Chapter 9, Finite Element Formulations of Solid Continua, makes a major effort to relate and develop the nonlinear strains defined by the Lagrangian and Eulerian coordinate systems. The formulation is shown characterizing the second Piola–Kirchhoff stress tensor and its relationship to the first Piola–Kirchhoff stress tensor and Cauchy stress tensor. The updated Lagrangian system from a stress-and-strain point of view is presented. This is one of the few publications to carry such a development out in an understandable fashion. The finite element formulation for both the total Lagrangian and updated Lagrangian is shown in a weak form with complete equations. As in all of the chapters, pertinent FORTRAN coding is shown in which significant subroutines are considered. Beam plate and shell examples are presented in which the nonlinear solutions are compared to the linear solutions and in certain cases to experimentations.

In Chapter 10, Material Nonlinearities and Coupled Problems, the author presents the development of relationships for material nonlinearity as opposed to geometric nonlinearity, which was covered in the preceding chapter. A presentation that can be found in several of the listed references is followed. The author then develops and presents a solution to a problem dominated by shear viscosity in the context of a power-law fluid using a finite element solution within an Eulerian system. The final development is related to the coupling of fluid flow with heat transfer. Chapter 7 relationships are presented in a coupling fashion in which the Navier–Stokes equations are combined with the heat transfer expressions (the energy equation). A sample problem is shown so that the ideas are illustrated. An appendix lists several algorithms to solve linear and nonlinear relations. Included are two schemes that are somewhat unique in any publication, the Riks and modified Riks techniques. Both these schemes can handle limit point characteristics, and the author explains them well.

It is this reviewer's opinion that J. N. Reddy has captured the essence of the field of nonlinear continuum mechanics in this textbook. The potential readership would be graduate students and researchers involved in constructing solutions to problems inherent to the nonconservative world, and there are many of these problems out there. The mechanics profession, because of computer capabilities, can not only consider these problems but also carry out their solutions.

Anthony N. Palazotto
Air Force Institute of Technology